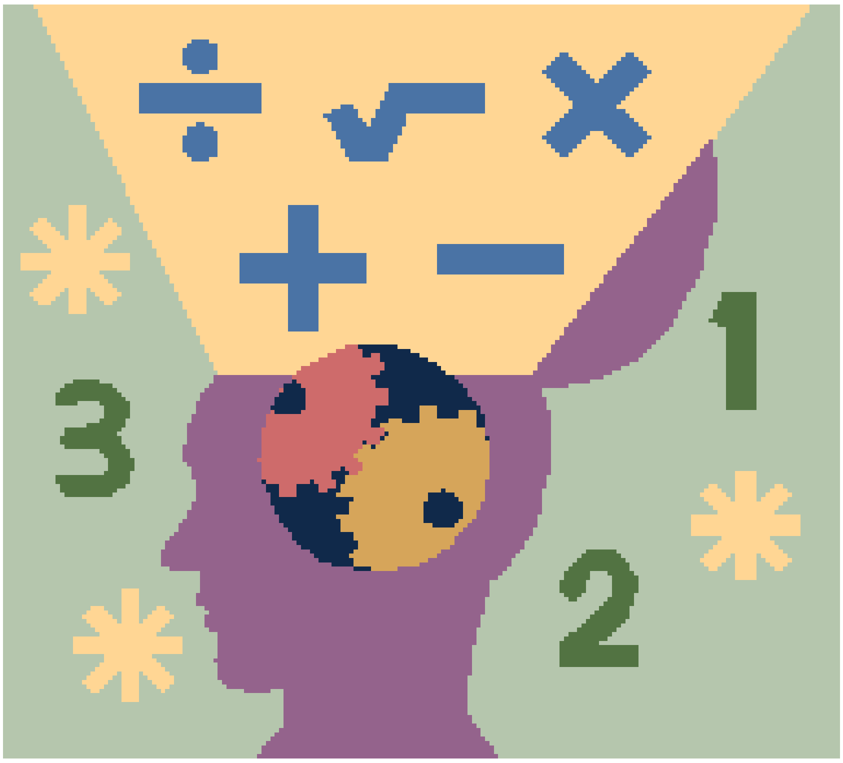
The Royal High School



Numeracy Booklet

A guide for S1 and S2 pupils, parents and staff

Introduction

What is the purpose of the booklet?

This booklet has been produced to give guidance to pupils and staff on how certain common Numeracy topics are taught in mathematics and throughout the school. Staff from all departments have been consulted during its production. It is hoped that using a consistent approach across all subjects will make it easier for pupils to progress.

How can it be used?

The booklet includes the Numeracy skills useful in subjects other than mathematics. For help with mathematics topics, pupils should refer to their mathematics textbook or ask their teacher for help.

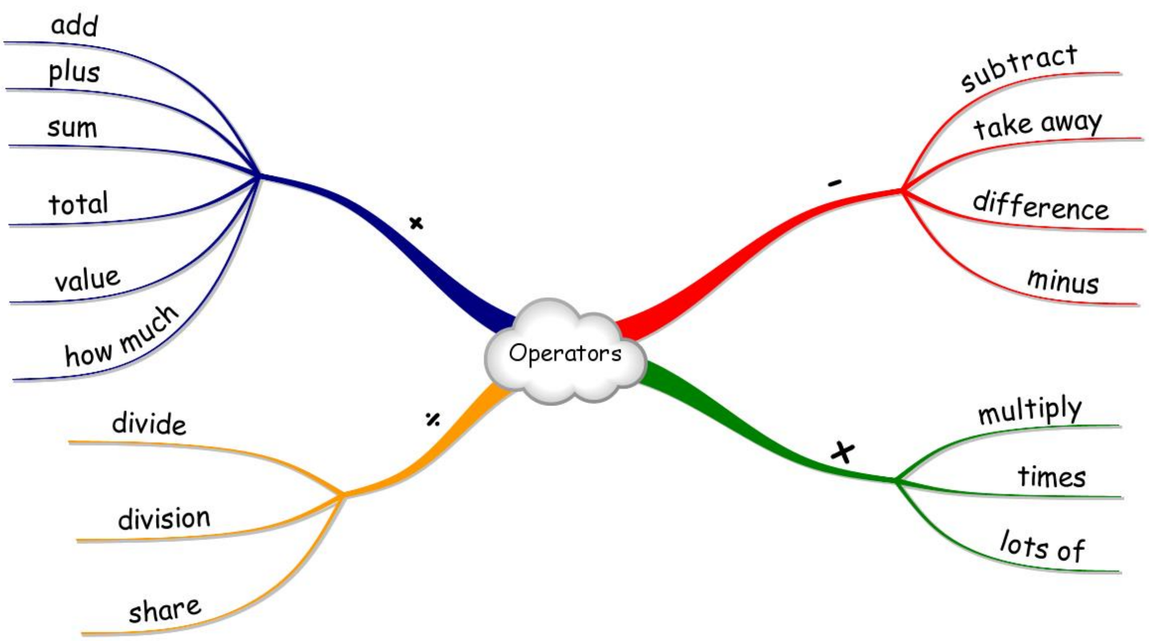
Why do some topics include more than one method?

In some cases (e.g. percentages), the method used will be dependent on the level of difficulty of the question, and whether or not a calculator is permitted.

For mental calculations, pupils should be encouraged to develop a variety of strategies so that they can select the most appropriate method in any given situation.

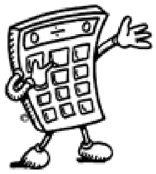
Table of Contents

Operators



	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Units



Here are some useful unit conversions:

10 mm	→	1 cm
100 cm	→	1 m
1000 m	→	1 km
1000 mg	→	1 g
1000 g	→	1 kg
1000 kg	→	1 tonne
1000 ml	→	1 litre
1 ml	→	1 cm^3
60 seconds	→	1 minute
60 minutes	→	1 hour
24 hours	→	1 day
7 days	→	1 week
14 days	→	1 fortnight
12 months	→	1 year
52 weeks	→	1 year
365 days	→	1 year
366 days	→	1 leap year
Decade	→	10 years
Century	→	100 years
Millennium	→	1000 years
1000	→	1 thousand
1000000	→	1 million
1000000000000	→	1 billion

Mathematical Dictionary (Key words):

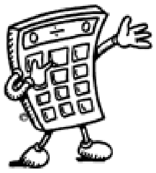
a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Axis	A line along the base or edge of a graph. Plural – Axes
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Compound Interest	Interest paid on the full balance of the account.
Data	A collection of information (may include facts, numbers or measurements).
Denominator	The bottom number in a fraction
Digit	A number
Discount	The amount an item is reduced by.
Equivalent fractions	Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than (>)	Is bigger or more than. Example: 10 is greater than 6 10 > 6
Gross Pay	Pay before deductions.
Histogram	A bar chart for continuous numerical values.
Increase	A value that has gone up.
Least	The lowest number in a group (minimum).
Less than (<)	Is smaller or lower than. Example: 15 is less than 21. 15 < 21.

Mathematical Dictionary (Key words):

Maximum	The largest or highest number in a group.
Mean	The average of a set of numbers
Median	A type of average - the middle number of an ordered set of data (ordered from lowest to highest)
Minimum	The smallest or lowest number in a group.
Mode	Another type of average – the most frequent number or category
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72 (the answers to the times tables)
Negative Number	A number less than zero. Shown by a negative sign. Example -5 is a negative number.
Net Pay	Pay after deductions.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1 ,3 ,5 ,7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BODMAS
Per annum	Each year (annually).
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight).
Prime Number	A prime number is a number that has exactly 2 factors (can only be divided by itself and 1). Note: 1 is **not** a prime number as it only has 1 factor.
Remainder	The amount left over when dividing a number.
Simple Interest	Interest paid only on an initial amount of money.
V.A.T.	Value Added Tax.

Addition

Mental strategies



There are a number of useful mental strategies for addition. Some examples are given below.

Example Calculate 54 + 27

Method 1 Add tens, then add units, then add together

50 + 20 = 70 4 + 7 = 11 70 + 11 = 81

Method 2 Split up number to be added into tens and units and add separately.

54 + 20 = 74 74 + 7 = 81

Method 3 Round up to nearest 10, then subtract

54 + 30 = 84 but 30 is 3 too much so subtract 3;
84 - 3 = 81

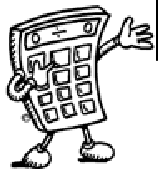
Written Method

When adding numbers, ensure that the numbers are lined up according to place value. Start at right hand side, write down units, carry tens.

Example Add 3032 and 589

3032		3032		3032		3032
+589	→	+589	→	+589	→	+589
1		21		621		3621
2 + 9 = 11		3+8+1=12		0+5+1=6		3 + 0 = 3

Subtraction



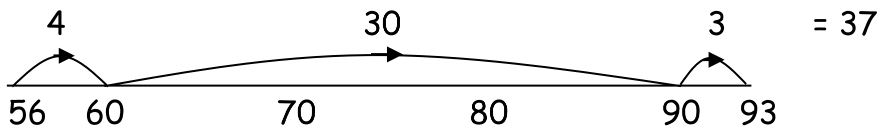
We use decomposition as a written method for subtraction (see below). Alternative methods may be used for mental calculations.

Mental Strategies

Example Calculate 93 - 56

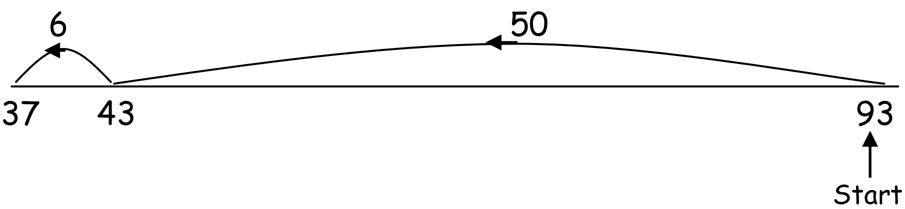
Method 1 Count on

Count on from 56 until you reach 93. This can be done in several ways e.g.



Method 2 Break up the number being subtracted

e.g. subtract 50, then subtract 6 93 - 50 = 43
43 - 6 = 37



Written Method

Example 1 4590 – 386

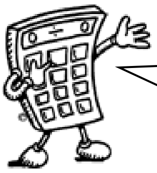
$$\begin{array}{r} {\scriptstyle 8\ 1} \\ 45\cancel{9}0 \\ -\ 386 \\ \hline 4204 \\ \hline \end{array}$$

Remember to "exchange" when you don't have enough.

Example 2 Subtract 692 from 14597

$$\begin{array}{r} {\scriptstyle 3\ 1} \\ 1\cancel{4}597 \\ -\ 692 \\ \hline 13905 \\ \hline \end{array}$$

Multiplication 1



It is essential that you know all of the multiplication tables from 1 to 12. These are shown in the tables square below.

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Mental Strategies

Example Find 39 x 6

Method 1

30 x 6
= 180

9 x 6
= 54

180 + 54
= 234

Method 2

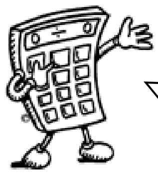
40 x 6
=240

40 is 1 too many
so take away 6x1

240 - 6
= 234

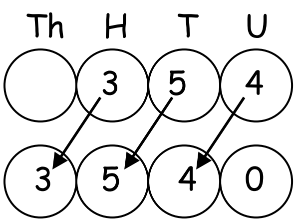
Multiplication 2

Multiplying by multiples of 10 and 100

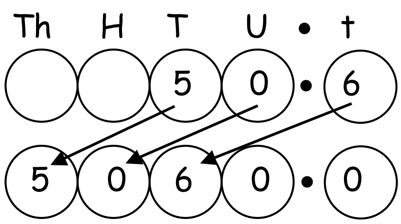


To multiply by **10** you move every digit ***one*** place to the left.
To multiply by **100** you move every digit ***two*** places to the left.
We do <u>**NOT**</u> just add a zero to the end of the number

Example 1 (a) Multiply 354 by 10 (b) Multiply 50.6 by 100



354 x 10 = 3540



50.6 x 100 = 5060

(c) 35 x 30

To multiply by 30, multiply by 3, then by 10.

35 x 3 = 105
105 x 10 = 1050

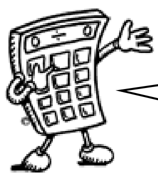
so 35 x 30 = 1050

(d) 436 x 600

To multiply by 600, multiply by 6, then by 100.

436 x 6 = 2616
2616 x 100 = 261600

so 436 x 600 = 261600



We may also use these rules for multiplying decimal numbers.

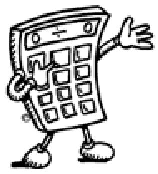
Example 2 (a) 2.36 x 20

2.36 x 2 = 4.72
4.72 x 10 = 47.2
so
2.36 x 20 = 47.2

(b) 38.4 x 50

38.4 x 5 = 192.0
192.0x 10 = 1920
so
38.4 x 50 = 1920

Division



You should be able to divide by a single digit or by a multiple of 10 or 100 without a calculator.

Written Method

Example 1 There are 192 pupils in first year, shared equally between 8 classes. How many pupils are in each class?

$$\begin{array}{r} 2\;4 \\ 8\overline{)1\;9^{3}2} \end{array}$$

There are 24 pupils in each class

Example 2 Divide 24 by 5

$$\begin{array}{r} 4\;r\;4 \\ 5\overline{)2^{2}4} \end{array}$$

Warning:

4 r 4 is <u>NOT</u> the same as 4.4

Example 3 Divide 4.74 by 3

$$\begin{array}{r} 1\,.\,5\;8 \\ 3\overline{)4\,.\,^{1}7^{2}4} \end{array}$$

When dividing a decimal number by a whole number, the decimal points must stay in line.

Example 4 A jug contains 2.2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

$$\begin{array}{r} 0\,.\,2\;7\;5 \\ 8\overline{)2\,.\,^{2}2^{6}0^{4}0} \end{array}$$

Each glass contains 0.275 litres

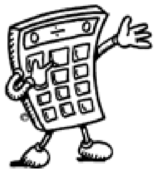
Where appropriate:
If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.

Order of Calculation (BODMAS)

Consider this: What is the answer to $2 + 5 \times 8$?

Is it $7 \times 8 = 56$ or $2 + 40 = 42$?

The correct answer is 42.



Calculations which have more than one operation need to be done in a particular order. The order can be remembered by using the mnemonic **BODMAS**

The **BODMAS** rule tells us which operations should be done first.
BODMAS represents:

(B)rackets
(O)f
(D)ivide
(M)ultiply
(A)dd
(S)ubract

Scientific calculators use this rule, some basic calculators may not, so take care in their use.

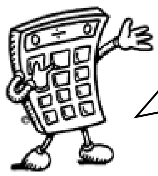
Example 1 $15 - 12 \div 6$ BODMAS tells us to divide first
$= 15 - 2$
$= 13$

Example 2 $(9 + 5) \times 6$ BODMAS tells us to work out the brackets first
$= 14 \times 6$
$= 84$

Example 3 $18 + 6 \div (5-2)$ Brackets first
$= 18 + 6 \div 3$ Then divide
$= 18 + 2$ Now add
$= 20$

RHS/0410

Evaluating Formulae



To find the value of a variable in a formula, we must substitute all of the given values into the formula, then use BODMAS rules to work out the answer.

Example 1

Use the formula $P = 2L + 2B$ to evaluate P when $L = 12$ and $B = 7$.

$P = 2L + 2B$	Step 1: write formula
$P = 2 \times 12 + 2 \times 7$	Step 2: substitute numbers for letters
$P = 24 + 14$	Step 3: start to evaluate (BODMAS)
$P = 38$	Step 4: write answer

Example 2

Use the formula $I = \frac{V}{R}$ to evaluate I when $V = 240$ and $R = 40$

$$I = \frac{V}{R}$$
$$I = \frac{240}{40}$$
$$I = 6$$

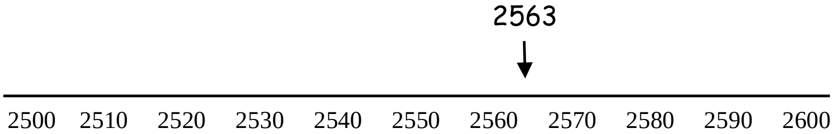
Example 3

Use the formula $F = 32 + 1.8C$ to evaluate F when $C = 20$

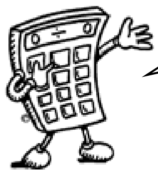
$$F = 32 + 1.8C$$
$$F = 32 + 1.8 \times 20$$
$$F = 32 + 36$$
$$F = 68$$

Rounding

Numbers can be rounded to give an approximation.



25**6**3 rounded to the nearest 10 is 2560.
2**5**63 rounded to the nearest 100 is 2600.
2563 rounded to the nearest 1000 is 3000.



When rounding numbers which are exactly in the middle, convention is to **round up**.
78**6**5 rounded to the nearest 10 is 78**7**0.

In general, to round a number, we must first identify the place value to which we want to round. We must then look at the next digit to the right (the "check digit") - if it is 5 or more round up.

Example 1 Round 1.54359 to 1 decimal place

The first number after the decimal point is a 5 - the check digit (the second number after the decimal point) is a 4, so leave the 5 as it is.

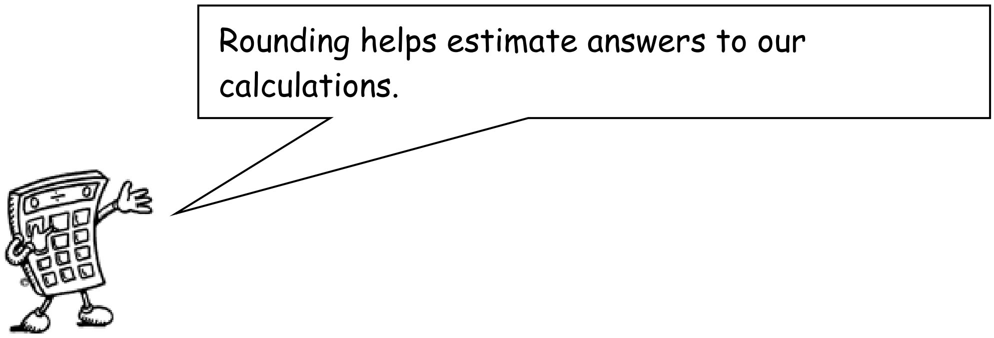
1.5 4 ~~359~~
= 1.5 (to 1 decimal place)

Example 2 Round 4.78632 to 2 decimal places

The second number after the decimal point is an 8 – the check digit (the third number after the decimal point) is a 6, so we round the 8 up to a 9

4.78 6 ~~32~~
= 4.79

Estimation : Calculation



Example 1
The number of computers sold over 4 days was recorded in the table below. How many computers were sold in total?

Monday	Tuesday	Wednesday	Thursday
486	205	197	321

Estimate = 500 + 200 + 200 + 300 = 1200

Calculate:

$$\begin{array}{r} 486 \\ 205 \\ 197 \\ +321 \\ \hline 1209 \\ \hline \end{array}$$

Answer = 1209 computers

Example 2
A muesli bar weighs 42g. There are 48 muesli bars in a box. What is the total weight of bars in the box?

Estimate = 50 x 40 = 2000g

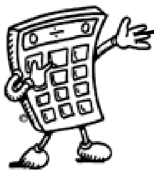
Calculate:

$$\begin{array}{r} 42 \\ \times 48 \\ \hline 336 \\ 1680 \\ \hline 2016 \\ \hline \end{array}$$

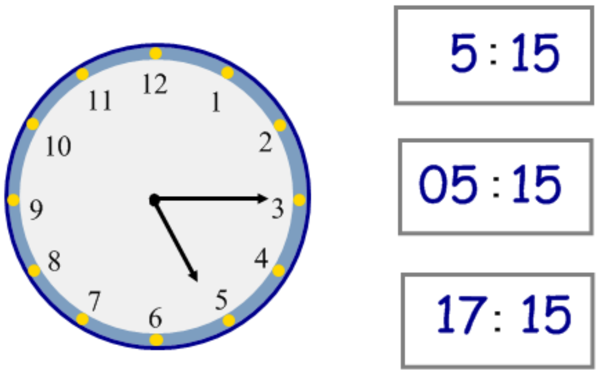
Answer = 2016g

Time

Time may be expressed in 12 hour clock or 24 hour clock.



Time can be displayed in many different ways.

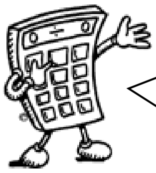


All these clocks show fifteen minutes past five, or quarter past five.

12 hour clock

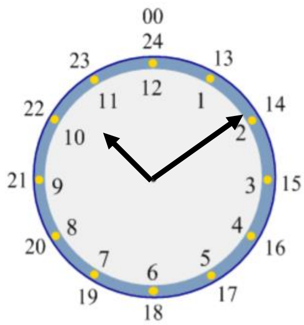
When writing times in 12 hour clock, we **need** to add a.m. or p.m. after the time.
a.m. is used for times between midnight and 12 noon (morning)
p.m. is used for times between 12 noon and midnight (afternoon / evening).

Time



In 24 hour clock:
The hours are written as numbers between 00 and 24
After 12 noon, the hours are numbered 13,14,15..etc
Midnight is expressed as 0000
We do **not** use am or pm with 24 hour clock

24 hour clock



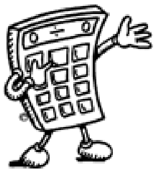
Examples

12 hour		24 hour
2:16 am	→	0216
8:55 am	→	0855
3:35 pm	→	1535
8:45 pm	→	2045
12:20 am	→	0020

Time Facts

60 seconds	→	1 minute
60 minutes	→	1 hour
24 hours	→	1 day

Time Facts



It is essential to know the number of months, weeks and days in a year, the number of days in each month.

Time Facts

The number of days in each month can be remembered using the rhyme:

"30 days has September,
April, June and November,
All the rest have 31,
Except February alone,
Which has 28 days clear,
And 29 in each leap year."

In 1 year, there are:
365 days (366 in a leap year)
52 weeks
12 months

Distance, Speed and Time.

For any given journey, the distance travelled depends on the speed and the time taken. If speed is constant, then the following formulae apply:

$$\text{Distance} = \text{Speed} \times \text{Time} \quad \text{or} \quad D = S\,T$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \quad \text{or} \quad S = \frac{D}{T}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \quad \text{or} \quad T = \frac{D}{S}$$

Example

Calculate the speed of a train which travelled 450 km in 5 hours

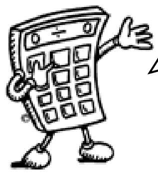
$$S = \frac{D}{T}$$

$$S = \frac{450}{5}$$

$$S = 90 \text{ km/h}$$

[In science speed is referred to as velocity]

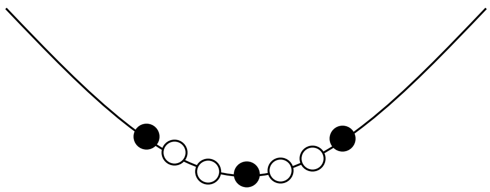
Fractions



Addition, subtraction, multiplication and division of fractions are studied in mathematics.
However, the examples below may be helpful in all subjects.

Understanding Fractions

Example
A necklace is made from black and white beads.

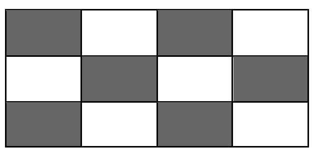


What fraction of the beads are black?

There are 3 black beads out of a total of 7, so $\frac{3}{7}$ of the beads are black.

Equivalent Fractions

Example
What fraction of the flag is shaded?



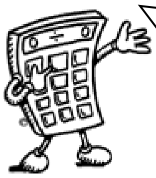
6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**.

Fractions

Simplifying Fractions



The top of a fraction is called the **numerator**, the bottom is called the **denominator**.
To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the same number.

$$\frac{\textbf{numerator}}{\textbf{denominator}}$$

Example 1

(a)
$$\frac{20}{25} \overset{\div 5}{\underset{\div 5}{=}} \frac{4}{5}$$

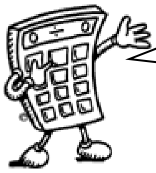
(b)
$$\frac{16}{24} \overset{\div 8}{\underset{\div 8}{=}} \frac{2}{3}$$

This can be done repeatedly until the numerator and denominator are the smallest possible numbers - the fraction is then said to be in its **simplest form**.

Example 2 Simplify $\frac{72}{84}$

$$\frac{72}{84} \overset{\div 2}{\underset{\div 2}{=}} \frac{36}{42} \overset{\div 2}{\underset{\div 2}{=}} \frac{18}{21} \overset{\div 3}{\underset{\div 3}{=}} \frac{6}{7} \text{ (simplest form)}$$

Calculating Fractions of a Quantity



To find the fraction of a quantity:
Divide by the denominator, multiply by the numerator.
[÷ by bottom, × by top]

Example 1 Find $\frac{1}{5}$ of £150

$\frac{1}{5}$ of £150 = 150 ÷ 5 x 1

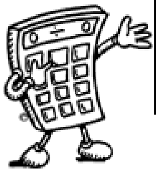
= £30

Example 2 Find $\frac{3}{4}$ of 48

$\frac{3}{4}$ of 48 = 48 ÷ 4 x 3

= 36

Percentage Facts



Percent means out of 100.
A percentage can be converted to an equivalent fraction or decimal.
The symbol for percent is **%**

36% means $\frac{36}{100}$

36% is therefore equivalent to $\frac{9}{25}$ and 0.36

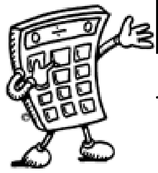
Common Percentages

Some percentages are used very frequently. It is useful to know these as fractions and decimals.

Percentage	Fraction (in simplest form)	Decimal
1%	$\frac{1}{100}$	0.01
10%	$\frac{10}{100} = \frac{1}{10}$	0.1
20%	$\frac{20}{100} = \frac{1}{5}$	0.2
25%	$\frac{25}{100} = \frac{1}{4}$	0.25
50%	$\frac{50}{100} = \frac{1}{2}$	0.5
75%	$\frac{75}{100} = \frac{3}{4}$	0.75
100%	$\frac{100}{100}$	1.0

Finding a Percentage of a Quantity
Non-Calculator

There are many ways to calculate percentages of a quantity. Some of the common ways are shown below.



Non- Calculator Methods

Method 1 Using Equivalent Fractions

Example Find 25% of £640

25% of £640 $= \frac{1}{4}$ of £640

$= 640 \div 4 \times 1$

= £160

Method 2 Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example Find 9% of 200g

1% of 200 $= \frac{1}{100}$ of 200

$= 200 \div 100 \times 1$

$= 2$

so 9% of 200g $= 9 \times 2 = 18$g

Method 3 Using 10%

This method is similar to the one above. First find 10% (by dividing by 10), then multiply to give the required value.

Example Find 70% of £35

10% of £35 $= \frac{1}{10}$ of 35

$= 35 \div 10 \times 1$

$= 3.50$

so 70% of £35 $= 7 \times 3.50 =$ £24.50

Finding a Percentage of a Quantity

Non- Calculator Methods (continued)

The previous 2 methods can be combined so as to calculate any percentage.

Example Find 23% of £15000

10% of £15000	= 1500
so 20%	= 1500 x 2
	= £3000
1% of £15000	= 150
so 3%	= 150 x 3
	= £450
23% of £15000	= £3000 + £450
	= £3450

Finding VAT (without a calculator)

Value Added Tax (VAT) = 15%
To find VAT, firstly find 10%

Example Calculate the total price of a computer which costs £650 excluding VAT

10% of £650	= 65	(divide by 10)
5% of £650	= 32.50	(divide previous answer by 2)
so 15% of £650	= 65 + 32.50	
	= £97.50	
Total price	= 650 + 97.50	
	= £747.50	

Finding a Percentage of a Quantity
Calculator Method

Calculator Method

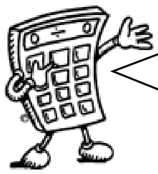
To find the percentage of a quantity using a calculator, change the percentage to a fraction.

Example 1 Find 23% of £15000

23% of 15000 $= \frac{23}{100} \times 15000$ or $= 0.23 \times 15000$

$= 15000 \div 100 \times 23$ $= £3450$

$= £3450$



We **<u>NEVER</u>** use the % button on calculators. The methods taught in the mathematics department are all based on converting percentages to fractions.

Example 2 House prices increased by 19% over a one year period. What is the new value of a house which was valued at £236000 at the start of the year?

$19\% = \frac{19}{100}$ so

Increase $= \frac{19}{100} \times 236000$ or $= 0.19 \times 236000$

$= 236000 \div 100 \times 19$ $= 44840$

$= 44840$

Value at end of year = original value + increase

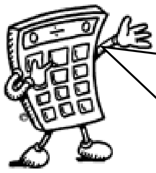
= 236000 + 44840

= 280840

The new value of the house is £280840

Finding a Percentage

Finding the percentage



To find a percentage of a total:

1. make a fraction,
2. change to a decimal by dividing the top by the bottom.
3. multiply by 100 to make a %

Example 1 There are 30 pupils in a class. 18 are girls. What percentage of the class are girls?

$$\frac{18}{30} = 18 \div 30$$
$$= 0.6$$
$$= 0.6 \times 100$$
$$= 60\%$$

60% of the class are girls

Example 2 James scored 36 out of 44 his biology test. What is his percentage mark?

$$\text{Score} = \frac{36}{44}$$
$$= 36 \div 44$$
$$= 0.81818\ldots$$
$$= 0.81818 \times 100$$
$$= 81.818..\%$$
$$= 82\% \text{ (to nearest whole number)}$$

Example 3 In a class, 14 pupils had brown hair, 6 pupils had blonde hair, 3 had black hair and 2 had red hair. What percentage of the pupils were blonde?

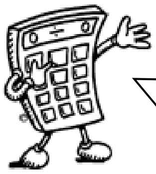
Total number of pupils = 14 + 6 + 3 + 2 = 25

6 out of 25 were blonde, so,

$$\frac{6}{25} = 6 \div 25$$
$$= 0.24$$
$$= 0.24 \times 100$$
$$= 24\%$$

24% of pupils were blonde.

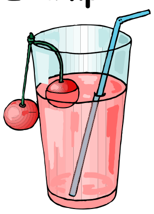
Ratio 1



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

Writing Ratios

Example 1



To make a fruit drink:
4 parts water is mixed with 1 part of cordial.
The ratio of water to cordial is 4:1
(said "4 to 1")
The ratio of cordial to water is 1:4.

Order is important when writing ratios.

Example 2

In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.

The ratio of red : blue : green is 5 : 7 : 8

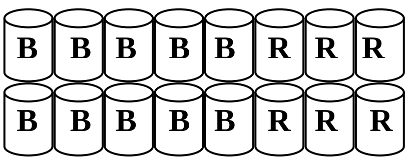
Simplifying Ratios

Ratios can be simplified in much the same way as fractions.

Example 1

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, each containing 5 tins of blue and 3 tins of red.



Blue : Red = 10 : 6
Blue : Red = 5 : 3

To simplify a ratio, divide each figure in the ratio by the **highest** common factor.

Ratio 2

Simplifying Ratios (continued)

Example 2
Simplify each ratio:
(a) 4:6 (b) 24:36 (c) 6:3:12

(a) 4:6
= 2:3

Divide each figure by 2

(b) 24:36
= 2:3

Divide each figure by 12

(c) 6:3:12
= 2:1:4

Divide each figure by 3

Example 3
Concrete is made by mixing 20 kg of sand with 4 kg cement. Write the ratio of sand : cement in its simplest form

Sand : Cement = 20 : 4
= 5 : 1

Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

Fruit	Nuts
3	2
15	**10**

$\times 5$ $\times 5$

So the chocolate bar will contain 10g of nuts.

Ratio 3

Sharing in a given ratio

Example

Lauren and Sean earn money by washing cars. By the end of the day they have made £90. As Lauren did more of the work, they decide to share the profits in the ratio 3:2. How much money did each receive?

Step 1 Add up the numbers to find the total number of parts

$3 + 2 = 5$

Step 2 Divide the total by this number to find the value of each part

$90 \div 5 = 18$

Step 3 Multiply each figure by the value of each part

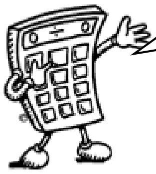
$3 \times 18 = 54$
$2 \times 18 = 36$

Step 4 Check that the total is correct

$54 + 36 = 90$ ✓

Lauren received £54 and Sean received £36

Proportion



When two quantities change in the same ratio, the quantities are said to be **directly proportional**.

It is often useful to make a table when solving problems involving proportion.

Example 1
A car factory produces 1500 cars in 30 days. How many cars would they produce in 90 days?

	Days	Cars	
×3	30	1500	×3
	90	**4500**	

The factory would produce 4500 cars in 90 days.

Example 2
5 adult tickets for the cinema cost £27.50. How much would 8 tickets cost?

	Tickets	Cost
	5	27.50
Find the cost of 1 ticket →	1	5.50
	8	44.00

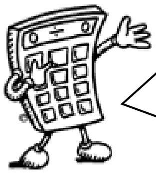
Working:

$$5 \overline{)27.50} = 5.50$$

$$5.50 \times 8 = 44.00$$

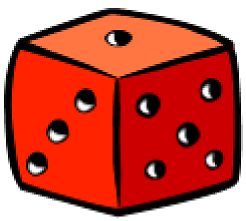
The cost of 8 tickets is £44

Probability



Probability is how likely or unlikely an event is of happening.
If an event is certain to happen, it has a probability of 1.
If an event is impossible or unlikely it has a probability of 0.
Probability of an event E happening:

$$P(E) = \frac{\textbf{number of ways an event can occur}}{\textbf{total number of different outcomes}}$$



A dice is rolled:

Example 1 What is the probability of rolling a 1?

$P(1) = \frac{1}{6}$

Example 2 What is the probability of rolling an even number?

3 even numbers

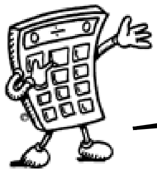
$P(\text{even}) = \frac{3}{6} = \frac{1}{2}$

Example 3 What is the probability of rolling a number greater than 4?

2 numbers greater than 4 (5 and 6)

$P(>4) = \frac{2}{6} = \frac{1}{3}$

Information Handling : Tables



It is sometimes useful to display information in graphs, charts or tables.

Example 1 The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

	J	F	M	A	M	J	J	A	S	O	N	D
Barcelona	13	14	15	17	20	**24**	27	27	25	21	16	14
Edinburgh	6	6	8	11	14	17	18	18	16	13	8	6

The average temperature in June in Barcelona is 24°C

Frequency Tables are used to present information. Often data is grouped in intervals.

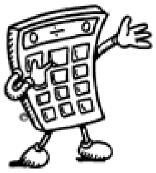
Example 2 Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

Mark	Tally	Frequency
16 - 20	||	2
21 - 25	~~||||~~ ||	7
26 - 30	~~||||~~ ||||	9
31 - 35	~~||||~~	5
36 - 40	|||	3
41 - 45	||	2
46 - 50	||	2

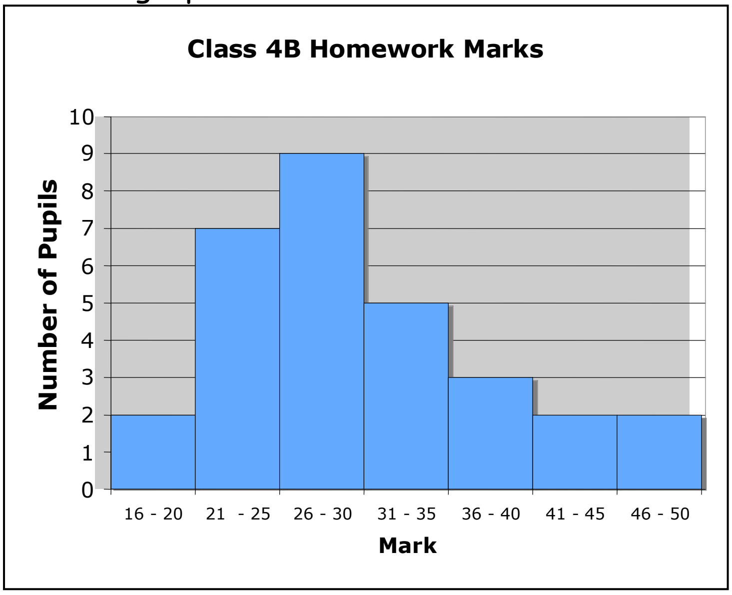
Each mark is recorded in the table by a tally mark.
Tally marks are grouped in 5's to make them easier to read and count.

Information Handling : Bar Graphs

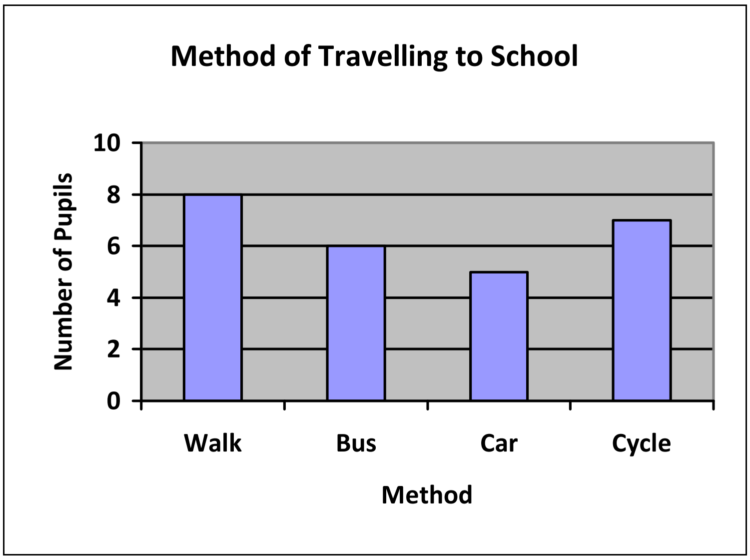


Bar graphs are often used to display data. The horizontal axis should show the categories or class intervals, and the vertical axis the frequency. All graphs should have a title, and each axis must be labelled.

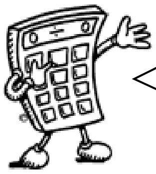
Example 1 The graph below shows the homework marks for Class 4B.



Example 2 How do pupils travel to school?

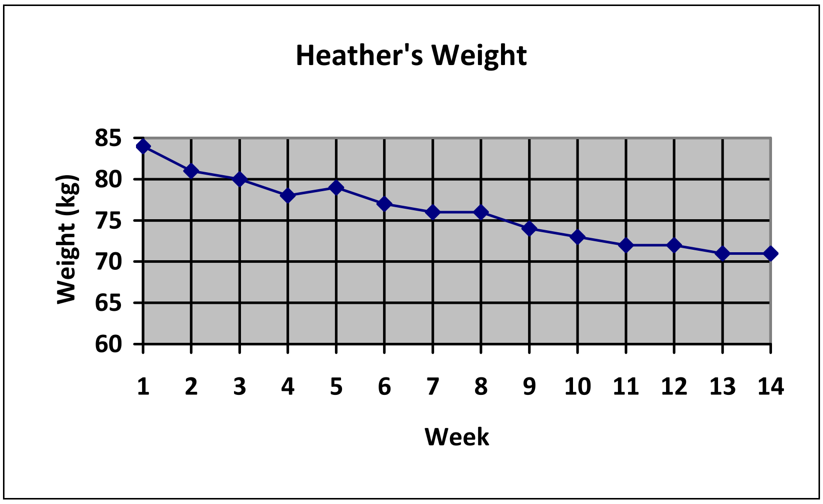


Information Handling : Line Graphs



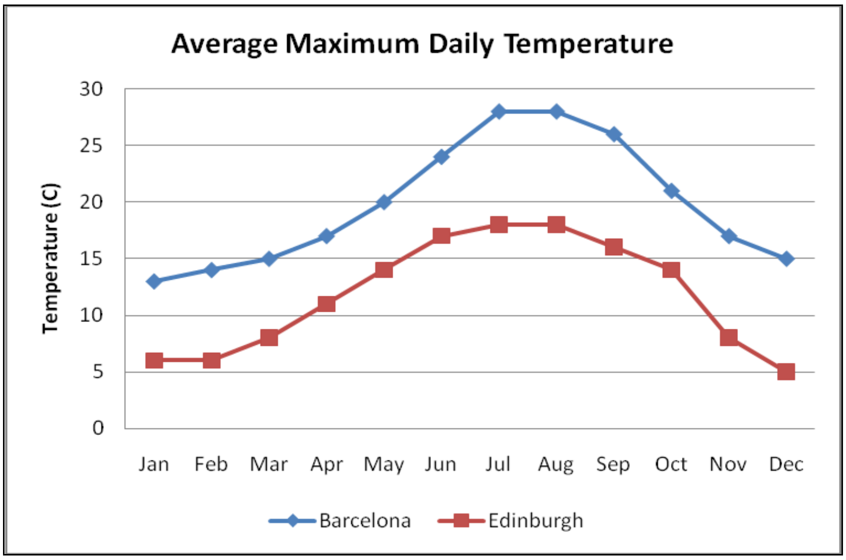
Line graphs consist of a series of points which are plotted, then joined by a line. All graphs should have a title, and each axis must be labelled. The trend of a graph is a general description of it.

Example 1 The graph below shows Heather's weight over 14 weeks as she follows an exercise programme.

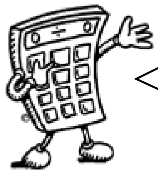


The trend of the graph is that her weight is decreasing.

Example 2 Graph of temperatures in Edinburgh and Barcelona.



Information Handling : Scatter Graphs

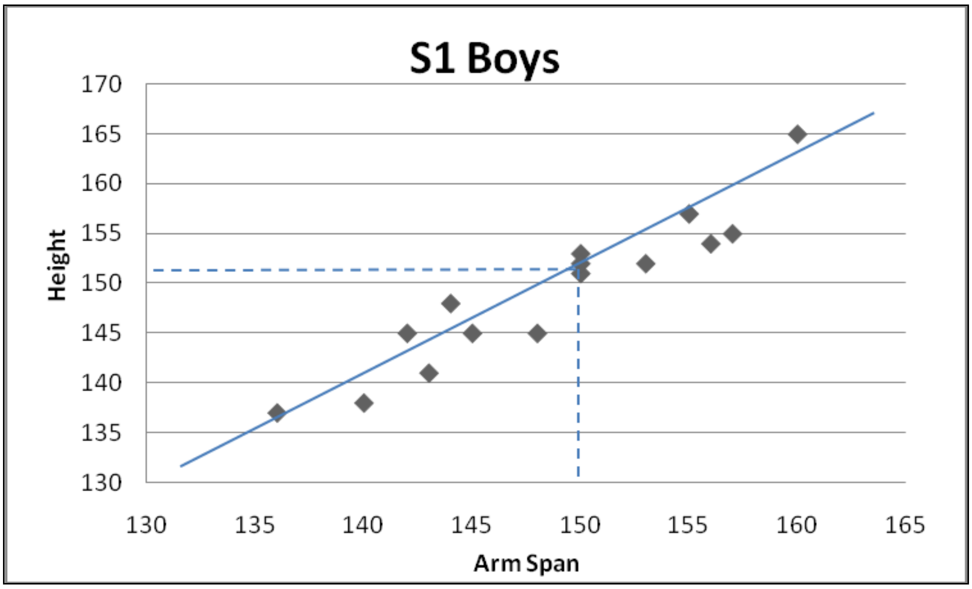


A scatter diagram is used to display the relationship between two variables.
A pattern may appear on the graph.
This is called a **correlation**.

Example The table below shows the height and arm span of a group of first year boys. This is then plotted as a series of points on the graph below.

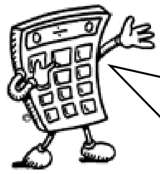
Arm Span (cm)	150	157	155	142	153	143	140	145	144	150	148	160	150	156	136
Height (cm)	153	155	157	145	152	141	138	145	148	151	145	165	152	154	137

The graph shows a general trend, that as the arm span increases, so does the height. This graph shows a positive correlation.



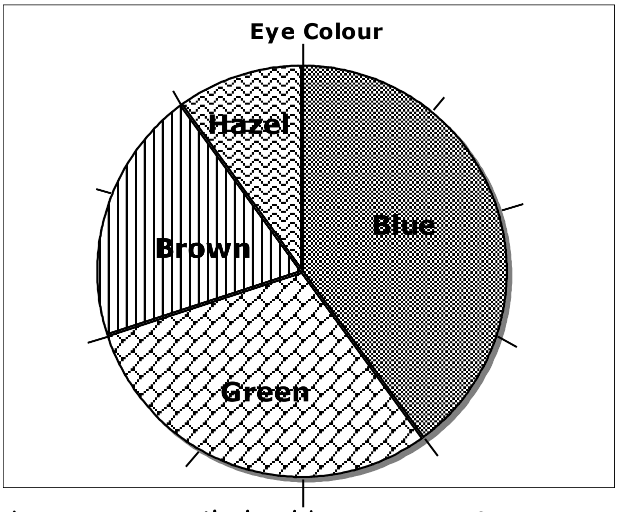
The line drawn is called the line of best fit. This line can be used to provide estimates. For example, a boy of arm span 150cm would be expected to have a height of around 151cm.

Information Handling : Pie Charts



A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Example 30 pupils were asked the colour of their eyes. The results are shown in the pie chart below.



How many pupils had brown eyes?
The pie chart is divided up into ten parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.

$\frac{2}{10}$ of 30 = 6 so 6 pupils had brown eyes.

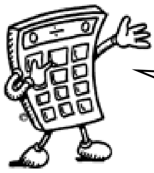
If no divisions are marked, we can work out the fraction by measuring the angle of each sector.

The angle in the brown sector is 72°.
so the number of pupils with brown eyes
$= \frac{72}{360} \times 30 = 6$ pupils.

If you find a value for each sector, this should add up to 30 pupils.

Information Handling : Pie Charts 2

Drawing Pie Charts



On a pie chart, the size of the angle for each sector is calculated as a fraction of 360°.

Example: In a survey about television programmes, a group of people were asked what was their favourite soap. Their answers are given in the table below. Draw a pie chart to illustrate the information.

Soap	Number of people
Eastenders	28
Coronation Street	24
Emmerdale	10
Hollyoaks	12
None	6

Total number of people = 80

Eastenders $= \frac{28}{80} \rightarrow \frac{28}{80} \times 360° = 126°$

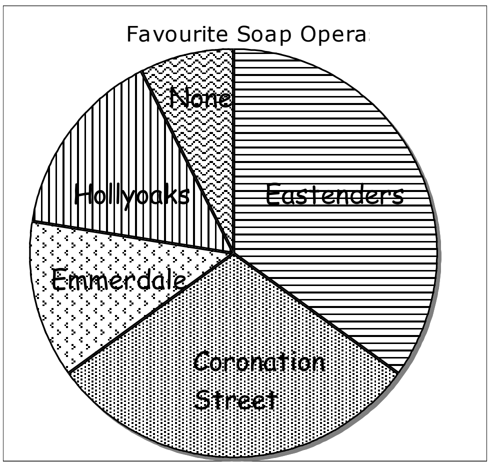
Coronation Street $= \frac{24}{80} \rightarrow \frac{24}{80} \times 360° = 108°$

Emmerdale $= \frac{10}{80} \rightarrow \frac{10}{80} \times 360° = 45°$

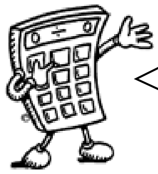
Hollyoaks $= \frac{12}{80} \rightarrow \frac{12}{80} \times 360° = 54°$

None $= \frac{6}{80} \rightarrow \frac{6}{80} \times 360° = 27°$

Check that the total = 360°



Information Handling : Averages



To provide information about a set of data, the average value may be given. There are 3 methods of finding the average value – the mean, the median and the mode.

Mean

The mean is found by adding all the data together and dividing by the number of values.

Median

The median is the middle value when all the data is written in numerical order from smallest to largest (if there are two middle values, the median is half-way between these values).

Mode

The mode is the value that occurs most often.

Range

The range of a set of data is a measure of spread.

Range = Highest value – Lowest value

Example A class scored the following marks for their homework assignment. Find the mean, median, mode and range of the results.

7, 9, 7, 5, 6, 7, 10, 9, 8, 4, 8, 5, 7, 10

Mean = $\frac{7+9+7+5+6+7+10+9+8+4+8+5+7+10}{14}$

= $\frac{102}{14} = 7.285...$

Mean = 7.3 to 1 decimal place

Ordered values: 4, 5, 5, 6, 7, 7, $\boxed{7,\ 7,}$ 8, 8, 9, 9, 10, 10
Median = 7

7 is the most frequent mark, so **Mode** = 7

Range = 10 – 4 = 6